

# LOCUS OF INDETERMINACY OF THE PRYM MAP

VITALY VOLOGODSKY

ABSTRACT. We provide an easy characterization for the locus of indeterminacy of the Prym map in terms of the dual graphs of stable curves. As a corollary, we show that the closure of the Fridman-Smith locus coincides with the locus of indeterminacy of the Prym map.

## 0. INTRODUCTION

This paper is intended to be an appendix to the [ABH] and here we are using notations and results of [ABH].

The extended Prym map is a rational map from the moduli space  $\bar{R}_g$  of stable curves  $C$  of genus  $2g - 1$  with an involution  $i$ , where the only base points of  $i$  are nodes and the involution does not exchange branches at that node, to the space  $\bar{A}_{g-1}^{vor}$ , the toroidal compactification of  $A_{g-1}$  for the 2nd Voronoi fan. On the dense open subset in  $\bar{R}_g$  corresponding to smooth curves it is given by associating to a pair  $(C, i)$  its Prym variety  $P(C, i)$ . Locus of indeterminacy of this map was described in [ABH] by a condition (\*) using the combinatorial data of the dual graph of a curve with an involution. This paper gives an easier description for the indeterminacy locus.

**Theorem 0.1.** *A curve with an involution is in the indeterminacy locus of the extended Prym map if and only if the curve is a degeneration of a Friedman-Smith example with the number of edges 4 or greater.*

Recall that the Friedman-Smith example with  $2n$  edges is a curve  $(C, i) \in \bar{R}_g$ , such that the curve  $C = C_1 \cup C_2$  is the union of two irreducible components, both invariant under the involution, intersecting in  $2n$  points, so that the involution is base point free and interchanges these  $2n$  nodes.

For  $n \geq 2$  the closure of the Friedman-Smith locus with  $2n$  edges contains  $[\frac{g-n+1}{2}] + 1$  irreducible components. Curve  $C' = C/i$  is the union  $C' = C'_1 \cup C'_2$  of two irreducible curves of genera  $g'_1$  and  $g'_2$ , where  $g'_1 + g'_2 = g - n + 1$ . Irreducible components of the Friedman-Smith locus correspond to cases when  $g'_1$  and  $g'_2$  are  $(0, g - n + 1), (1, g - n), \dots, ([\frac{g-n+1}{2}], [\frac{g-n+2}{2}])$ .

Note that in our space  $\bar{R}_g$  there are no stable curves having double points with the 2 branches exchanged by the involution (called the nodes of type (2) in [ABH]).

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## 1. PRELIMINARY RESULTS

We will call a vertex of the dual graph a *bold vertex* if the corresponding component of the curve is fixed by the involution and we will call an edge a *bold edge* if the corresponding node is a node of type (1), see [ABH] for definition. We will call all other vertices and edges *ordinary*. Note that if we have a bold edge then its beginning and the end are bold vertices. Therefore, the union of bold vertices and edges forms a subgraph of the dual graph. We will denote this subgraph by  $B(\Gamma)$ .

Proof of one direction for the Theorem 0.1 is immediate.

**Proposition 1.1.** *If the curve is a degeneration of a Friedman-Smith example with at least 4 edges, then the curve is in the indeterminacy locus of the Prym map.*

*Proof.* Prym map is not defined for the Friedman-Smith examples, see [FS] or [ABH] Section 4.2.3. The indeterminacy locus for every rational map is closed, therefore degenerations of the Friedman-Smith examples are also in the indeterminacy locus.  $\square$

Let us consider the dual graphs which are not degenerations of Friedman-Smith examples with the number of edges at least 4.

**Lemma 1.2.** *If the dual graph  $\Gamma$  of a stable curve  $C$  with an involution contains two disjoint connected equivariant subgraphs  $\Gamma_1$  and  $\Gamma_2$  which are connected by at least 4 ordinary edges (so that the beginnings of these edges are in one subgraph and the ends are in the other), but are not connected by any bold path then this curve is a degeneration of a Friedman-Smith example with at least 4 edges.*

*Proof.* We will construct two disjoint equivariant subgraphs  $\Gamma'_1$  and  $\Gamma'_2$  such that together they contain all vertices of the graph  $\Gamma$  and every edge of  $\Gamma$  either belongs to one of the subgraphs  $\Gamma'_1$  or  $\Gamma'_2$  or is an ordinary edge which begins at one subgraph and ends at the other.

To find these  $\Gamma'_1$  and  $\Gamma'_2$  we will take  $\Gamma_1$  and  $\Gamma_2$  and start adding to them vertices and edges until we will get required  $\Gamma'_1$  and  $\Gamma'_2$ .

First let us consider connected components of the bold subgraph  $B(\Gamma)$ . Each bold component cannot intersect with both subgraphs  $\Gamma_1$  and  $\Gamma_2$ , otherwise we have a bold path connecting these subgraphs. We will change each  $\Gamma_i$  by the union of  $\Gamma_i$  and all components of  $B(\Gamma)$  which has non empty intersection with  $\Gamma_i$ . It is easy to see that the new  $\Gamma_1$  and  $\Gamma_2$  still satisfy all the conditions from the statement of the Lemma. In addition, the compliment  $\Gamma \setminus (\Gamma_1 \cup \Gamma_2)$  is attached to  $\Gamma_1 \cup \Gamma_2$  through the ordinary edges only.

Next let us consider the following topological space  $\Gamma \setminus (\Gamma_1 \cup \Gamma_2)$ . Connected components of this space can be of three different types:

- 1) components attached to  $\Gamma_1$  only,
- 2) components attached to  $\Gamma_2$  only,
- 3) components attached to both  $\Gamma_1$  and  $\Gamma_2$ .

We will change  $\Gamma_1$  by the union  $\Gamma_1 \cup$  (all components of type 1) and  $\Gamma_2$  by  $\Gamma_2 \cup$  (all components of type 2). Again,  $\Gamma_1$  and  $\Gamma_2$  still satisfy all the conditions from the statement of the Lemma.

Now, to get desired  $\Gamma'_1$  and  $\Gamma'_2$ , we can take  $\Gamma'_1$  to be equal  $\Gamma_1$ . For vertices of  $\Gamma'_2$  we will take all vertices of  $\Gamma$  which are not from subgraph  $\Gamma'_1$  and for edges of  $\Gamma'_2$  we will take all edges whose beginning and ends are vertices of  $\Gamma'_2$ .

Note that the number of edges connecting  $\Gamma'_1$  and  $\Gamma'_2$  is at least the same as the number of edges connecting original  $\Gamma_1$  and  $\Gamma_2$ , which is not less than 4.

Consider decomposition of the curve  $C = C_1 \cup C_2$  where each  $C_i$  correspond to subgraph  $\Gamma'_i$ . Each part  $C_i$  is invariant with respect to involution and can be obtained as a degeneration of smooth curves with involution. Then the whole curve  $C$  is a degeneration of the Friedman-Smith example and as we have noticed the number of nodes is at least 4.  $\square$

## 2. CONDITION (\*)

Let us fix an orientation on the graph  $\Gamma$  which is compatible with the involution  $i$ . Following [ABH] edges  $e_j$  give coordinate functions on  $C_1(\Gamma, \mathbb{R})$ . We have three possibilities:

1.  $z_j$  is identically zero on  $X^-$ .
2. Image  $z_j(X^-) = \mathbb{Z}$ .
3. Image  $z_j(X^-) = \frac{1}{2}\mathbb{Z}$ .

From [ABH] we have the following combinatorial condition

(\*) The linear functions  $m_j z_j$  define a dicing of the lattice  $X^-$ .

In this dicing we are not using functions of type 1,  $m_j = 1$  for the functions  $z_j$  of type 2 and  $m_j = 2$  for the functions  $z_j$  of type 3.

Another characterization of possibilities 2 and 3 for image  $z_j(X^-)$  is as follows. Image  $z_j(X^-) = \mathbb{Z}$  if for every simple oriented cycle  $\omega \in H^1(\Gamma, \mathbb{Z})$  in the graph  $\Gamma$   $\text{mult}_{e_j} \omega = 1$  implies  $\text{mult}_{e_{i(j)}} \omega = -1$ . Image  $z_j(X^-) = \frac{1}{2}\mathbb{Z}$  if there exists a simple oriented cycle  $\omega \in H^1(\Gamma, \mathbb{Z})$  with  $\text{mult}_{e_j} \omega = 1$  but  $\text{mult}_{e_{i(j)}} \omega = 0$ .

Let  $d$  be the dimension of the space  $X^- \otimes \mathbb{R}$ . Then condition (\*) is equivalent to the following:

If the intersection of  $d$  hyperplanes  $\{m_{j_k} z_{j_k} = n_{j_k}, n_{j_k} \in \mathbb{Z}, k = 1, \dots, d\}$  is 0-dimensional (i.e. a point), then it is in the lattice  $X^-$ .

It is enough to check this condition for the sets of  $\{n_{j_k}, k = 1, \dots, d\}$  where all except one  $n_{j_k}$  are 0 and the remaining  $n_{j_k}$  is 1. If the intersections of hyperplanes corresponding to these sets  $\{n_{j_k}, k = 1, \dots, d\}$  are in the lattice  $X^-$  then, since all other  $\{n_{j_k}, k = 1, \dots, d\}$  are integer linear combinations of those sets, their corresponding intersections of the hyperplanes are integer linear combinations of elements from  $X^-$ , and therefore belong to  $X^-$ .

We can reprove Proposition 1.1 without using the geometrical meaning of condition (\*).

*Proof.* Let curve  $C$  be a degeneration of the Friedman-Smith example with  $2n$  edges with  $n \geq 2$ . Thus curve  $C$  is a union  $C_1 \cup C_2$  where each  $C_j$  is a degeneration of a family of smooth curves and intersection  $C_1 \cap C_2$  consists of  $n$  pairs of exchanged nodes. Then in the dual graph for  $C$  there are two equivariant subgraphs  $\Gamma_1$  and  $\Gamma_2$  corresponding to  $C_1$  and  $C_2$  and these subgraphs  $\Gamma_1$  and  $\Gamma_2$  are connected by the  $n$  pairs of exchanging edges. We will denote them  $e_1, e'_1 = i(e_1), \dots, e_n, e'_n = i(e_n)$ . It is clear that these edges are of type 3. In definition of condition (\*) all  $m_j$  corresponding to  $e_1, e'_1, \dots, e_n, e'_n$  equal 2.

Now if we have an arbitrary simple cycle  $\omega$  then either it does not contain any of the edges  $e_1, e'_1, \dots, e_n, e'_n$  or it passes through even number of them. With counting multiplicities the sum  $\omega - i(\omega)$  passes through  $e_1, \dots, e_n$  the same even number of times. All the elements of the lattice  $X^-$  are of the form  $\frac{1-i}{2}(\sum_l k_l \omega_l)$  where  $k_l \in \mathbb{Z}$  and  $\omega_l$  are simple oriented

cycles. Therefore for any element of  $X^-$  we have  $\sum_{m=1}^n \text{mult}_{e_m}(\frac{1-i}{2}(\sum_l k_l \omega_l))$  is an integer. We can choose some  $d - n$  edges  $e_{j_{n+1}}, \dots, e_{j_d}$  with the functions  $z_{j_{n+1}}, \dots, z_{j_d}$  so that the intersection of  $d$  hyperplanes  $\{2z_1 = 1, 2z_2 = 0, \dots, 2z_n = 0, m_{j_{n+1}} z_{j_{n+1}} = 0, \dots, m_{j_d} z_{j_d} = 0\}$  is 0-dimensional. But the point of intersection  $x$  cannot be an element of  $X^-$  since  $\sum_{m=1}^n \text{mult}_{e_m}(x) = \frac{1}{2}$  is not an integer. Condition (\*) fails.  $\square$

To finish the proof of Theorem 0.1, let us prove the other direction in its statement.

Let us compare functions  $z_j$  and  $z_{i(j)}$  when restricted to  $X^-$ . Elements of  $X^-$  are of the form  $\frac{1-i}{2}(\sum_l k_l \omega_l)$  where  $k_l \in \mathbb{Z}$  and  $\omega_l$  are simple oriented

cycles. If  $\omega_l = \sum_{m=1}^{\text{length}_{\omega_l}} e_{l_m}$  then

$$\frac{1-i}{2}[\sum_l k_l \omega_l] = \frac{1}{2}[\sum_l k_l (\sum_{m=1}^{\text{length}_{\omega_l}} e_{l_m}) - \sum_l k_l (\sum_{m=1}^{\text{length}_{\omega_l}} i(e_{l_m}))].$$

Since the orientation of the graph  $\Gamma$  is compatible with the involution  $i$  we have  $i(e_{l_m}) = +e_{i(l_m)}$  and thus  $z_j = -z_{i(j)}$  when restricted on  $X^-$ .

Intersection of  $d$  hyperplanes  $\{m_{j_k}z_{j_k} = n_{j_k}, n_{j_k} \in \mathbb{Z}, k = 1, \dots, d\}$  is 0-dimensional if and only if the intersection of  $\{m_{j_k}z_{j_k} = 0, k = 1, \dots, d\}$  is  $0 \in X^- \otimes R$ . The later is equivalent to  $\{m_{j_k}z_{j_k} = 0, m_{j_k}z_{i(j_k)} = 0, k = 1, \dots, d\}$  having intersection  $0 \in X^- \otimes R$ . Note that  $z_j(\omega) = z_{i(j)}(\omega) = 0$  for an element  $\omega \in X^-$  is equivalent to the fact that  $\omega$  can be represented as  $\frac{1-i}{2}(\sum_l k_l \omega_l)$  where all  $\omega_l$  are simple cycles in  $\Gamma$  not passing through  $e_j$  or  $e_{i(j)}$ . Therefore intersection of  $\{m_{j_k}z_{j_k} = 0, m_{j_k}z_{i(j_k)} = 0, k = 1, \dots, n\}$  consist of the elements of  $X^-$  of the form  $\frac{1-i}{2}(\sum_l \omega_l)$  with  $\omega_l \in H^1(\Gamma \setminus (e_{j_1} \cup e_{i(j_1)} \cup \dots \cup e_{j_n} \cup e_{i(j_n)}), \mathbb{Z})$ . Finally, intersection of  $d$  hyperplanes  $\{m_{j_k}z_{j_k} = n_{j_k}, n_{j_k} \in \mathbb{Z}, k = 1, \dots, d\}$  is 0-dimensional if and only if for the graph  $\Gamma' = \Gamma \setminus (e_{j_1} \cup e_{i(j_1)} \cup \dots \cup e_{j_n} \cup e_{i(j_n)})$  all elements  $\omega \in H^1(\Gamma', \mathbb{Z})$  are invariant with respect to  $i$ .

In the case of 0-dimensional intersection let us look closely at the subgraph  $\Gamma'$ . It may have more than one connected component. But it is impossible that two of these components are exchanged by the involution. Indeed, for connected curve with involution dimension of the corresponding  $X^-$  can be computed as  $n_e - c_e$  where  $2n_e$  is the number of exchanged under the involution nodes (nodes of type (3)) (or the number of ordinary edges in the dual graph) and  $2c_e$  is the number of exchanged components (or the number of ordinary vertices). For the graph  $\Gamma$  we have  $n_e - c_e = d$ . Subgraph  $\Gamma'$  was obtained by erasing  $2d$  ordinary edges, thus for  $\Gamma'$  we have  $n_e - c_e = 0$ . Now  $X^-$  for  $\Gamma'$  is 0, so for every invariant under the involution connected component of  $\Gamma'$  we must have  $n_e - c_e = 0$ . Components of  $\Gamma'$  exchanged by  $i$  consist of ordinary vertices and ordinary edges only. To have  $X^- = 0$  for  $\Gamma'$  all exchanged components should have no cycles. Then they are trees and for each of them  $n_e - c_e = -1$ . If the exchanged components present in  $\Gamma'$  then for  $\Gamma'$   $n_e - c_e$  would be negative. All connected components of  $\Gamma'$  are fixed under  $i$ .

Now consider the point of intersection for the hyperplanes  $\{m_{j_1}z_{j_1} = 1, m_{j_k}z_{j_k} = 0, k = 2, \dots, d\}$ .

If the edge  $e_{j_1}$  has the beginning  $v_{j_1}$  and the end  $v'_{j_1}$  at the same connected component of  $\Gamma'$  then there is a path  $t$  inside of  $\Gamma'$  which starts at  $v'_{j_1}$  and ends at  $v_{j_1}$ . Cycle  $\omega = t + e_{j_1}$  has  $\text{mult}_{e_{j_1}} \omega = 1$  but  $\text{mult}_{e_{i(j_1)}} \omega = 0$  which means that  $m_{j_1} = 2$ . Now the element  $x^- = \frac{1-i}{2}(\omega) \in X^-$  satisfies  $\{m_{j_1}z_{j_1}(x^-) = 1, m_{j_k}z_{j_k}(x^-) = 0, k = 2, \dots, d\}$ . The point of intersection of the hyperplanes is in  $X^-$ .

If the edge  $e_{j_1}$  has the beginning  $v_{j_1}$  and the end  $v'_{j_1}$  in different components of  $\Gamma'$  then the edges  $e_{j_1}$  and  $e_{i(j_1)}$  start (resp. end) at the same component. There is a path  $t$  inside of  $\Gamma'$  which starts at  $v_{j_1}$  and ends at  $i(v_{j_1})$ . Similarly, there is a path  $t'$  inside of  $\Gamma'$  which starts at  $v'_{j_1}$  and ends at  $i(v'_{j_1})$ .  $\omega = e_{j_1} + t' - e_{i(j_1)} - t$  is a cycle in  $\Gamma$ . Element  $x^- = \frac{1-i}{2}(\omega) \in X^-$  satisfies  $\{z_{j_1}(x^-) = 1, z_{j_k}(x^-) = 0, k = 2, \dots, d\}$ . To see that  $x^-$  is the point

of intersection for the hyperplanes  $\{m_{j_1}z_{j_1} = 1, m_{j_k}z_{j_k} = 0, k = 2, \dots, d\}$  we need to show that  $m_{j_1} = 1$ .

We will show that if there exists a simple oriented cycle  $\omega \in H^1(\Gamma, \mathbb{Z})$  with  $\text{mult}_{e_{j_1}} \omega = 1$  but  $\text{mult}_{i(e_{j_1})} \omega = 0$  then  $\Gamma$  is a degeneration of Friedman-Smith example with at least 4 edges. Cycle  $\omega$  goes along some of the edges  $e_{j_k}$  and  $e_{i(j_k)}$  and passes through some connected components  $C_1, \dots, C_l$  of the subgraph  $\Gamma'$ . We can assume that the edge  $e_{j_1}$  starts at  $C_1$  and ends at  $C_2$ . Cycle  $\omega$  is simple so it cannot reenter  $C_1$  along  $e_{j_1}$ .  $\text{mult}_{i(e_{j_1})} \omega = 0$ , so  $\omega$  cannot reenter  $C_1$  along  $i(e_{j_1})$ . There is another edge  $e'$  through which  $\omega$  reenters  $C_1$ . Let  $\Gamma_1 = C_1$ ,  $\Gamma'_2 = (C_2 \cup \dots \cup C_l) \cup (\omega \setminus (C_1 \cup e_{j_1} \cup e'))$  and  $\Gamma_2 = \Gamma'_2 \cup i(\Gamma'_2)$ . Subgraphs  $\Gamma_1$  and  $\Gamma_2$  are connected by at least 4 ordinary edges:  $e_{j_k}$ ,  $e_{i(j_k)}$ ,  $e'$  and  $i(e')$ . Therefore, they satisfy the assumptions of the Lemma 1.2 and graph  $\Gamma$  is a degeneration of Friedman-Smith example with at least 4 edges.

This proves Theorem 0.1.

### 3. CONDITION (\*\*)

In this section we will discuss condition (\*\*) of [ABH] which is stronger than condition (\*). When condition (\*) holds but (\*\*) fails we have a Prym variety corresponding to the double cover of stable curves but this Prym cannot be canonically embedded into the Jacobian of the covering curve.

From [ABH] we have the following definition of combinatorial condition (\*\*)

(\*\*)  $[\Delta^-]$  is a dicing with respect to the lattice  $2X^-$ .

In the notations of the previous section this condition (\*\*) is equivalent to the following:

If the intersection of  $d$  hyperplanes  $\{z_{j_k} = n_{j_k}, n_{j_k} \in \mathbb{Z}, k = 1, \dots, d\}$  is 0-dimencional (i.e. a point), then it is in the lattice  $2X^-$ .

The first condition of the following theorem gives an easier description for (\*\*).

**Theorem 3.1.** *The following three conditions are equivalent*

- (i) *Curve is not a degeneration of a Friedman-Smith example with the number of edges 2 or greater.*
- (ii) *Condition (\*) holds and there are no functions  $z_j$  of type (2).*
- (iii) *Condition (\*\*) holds.*

*Proof.* (i) implies (ii). If (\*) does not hold then (\*\*) does not hold either. We only need to show that if (\*) holds but the graph has an edge  $e_j$  with the corresponding function  $z_j$  of type (2) then the graph is a degeneration of a Friedman-Smith example with 2 edges.

Let  $v_j$  be the beginning of  $e_j$  and  $v'_j$  be the end of  $e_j$ . Let us consider connected components of  $\Gamma'_j = \Gamma \setminus (e_j \cup e_{i(j)})$ . The number of components

is at most 3. If there is only one component in  $\Gamma'_j$  then there is a path  $t$  in  $\Gamma'_j$  connecting  $v_j$  and  $v'_j$ . Thus for the simple cycle  $\omega = e_j - t$  we have  $\text{mult}_{e_j} \omega = 1$  and  $\text{mult}_{e_{i(j)}} \omega = 0$  which is impossible since  $e_j$  is an edge of type 2. If there were three components in  $\Gamma'_j$  then edges  $e_j$  and  $e_{i(j)}$  would have been of type 1. The only possibility is when  $\Gamma'_j$  has two components  $\Gamma'_{j1}$  and  $\Gamma'_{j2}$ . Since for every simple cycle  $\omega$  we have  $\text{mult}_{e_j} \omega = -\text{mult}_{e_{i(j)}} \omega$  then beginnings  $v_j$  and  $v'_j$  belong to the same component of  $\Gamma'_j$ , say  $\Gamma'_{j1}$ . Therefore  $\Gamma'_{j1}$  is invariant with respect of involution  $i$  and so is  $\Gamma'_{i2}$ . Now consider decomposition of the curve  $C = C_1 \cup C_2$  where each  $C_k$  correspond to subgraph  $\Gamma'_{jk}$ . Each part  $C_k$  is invariant with respect to involution and can be obtained as a degeneration of smooth curves with involution. Then the whole curve  $C$  is a degeneration of the Friedman-Smith example with the number of edges 2.

(ii) *implies (i)*. Assume that (i) fails. The curve  $C$  is a degeneration of a Friedman-Smith example with 2 edges. Thus curve  $C$  is a union  $C_1 \cup C_2$  where each  $C_j$  is a degeneration of a smooth curve and intersection  $C_1 \cap C_2$  consists of two exchanged nodes. Then in the dual graph for  $C$  there are two equivariant subgraphs  $\Gamma_1$  and  $\Gamma_2$  corresponding to  $C_1$  and  $C_2$  and these subgraphs  $\Gamma_1$  and  $\Gamma_2$  are connected by the pair of exchanging edges. It is clear that these edges are of type 2.

(ii) *implies (iii)*. Since there are no edges of type 2 then in definition of condition (\*) all  $m_j$  equal 2. Condition (\*) holds, so if the intersection of  $d$  hyperplanes  $\{2z_{j_k} = n_{j_k}, n_{j_k} \in \mathbb{Z}, k = 1, \dots, d\}$  is 0-dimensional, then it is in the lattice  $X^-$ . Thus intersection of  $\{2z_{j_k} = 2n_{j_k}, n_{j_k} \in \mathbb{Z}, k = 1, \dots, d\}$  is in  $2X^-$ , but this is the same point as intersection of  $\{z_{j_k} = n_{j_k}, n_{j_k} \in \mathbb{Z}, k = 1, \dots, d\}$ . Therefore condition (\*\*) holds.

(iii) *implies (ii)*. Let us take an arbitrary edge  $e_j$  and the corresponding function  $z_j$ . If  $z_j$  is nontrivial (i.e. not of type 1) then we can choose some  $d-1$  edges  $e_{j_2}, \dots, e_{j_d}$  with the functions  $z_{j_2}, \dots, z_{j_d}$  so that the intersection of  $d$  hyperplanes  $\{z_j = 1, z_{j_2} = 0, \dots, z_{j_d} = 0\}$  is 0-dimensional and by condition (\*\*) is some point  $2x^- \in 2X^-$ . Now  $x^- \in X^-$  and  $z_j(x^-) = \frac{1}{2}$ . Thus edge  $e_j$  and the corresponding function  $z_j$  are of type 3 and not of type 2.

□

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF GEORGIA, ATHENS, GA 30602  
 E-mail address: vologods@math.uga.edu